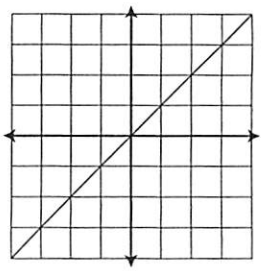
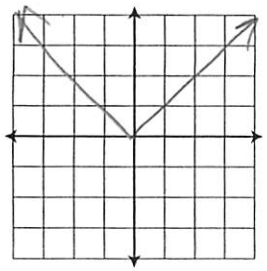
4.3 Exercise Set

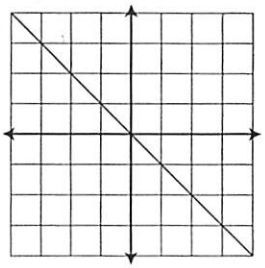
1. The graph of $f(x)$ is shown. Sketch the graph of $|f(x)|$.

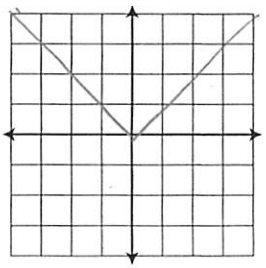
a)



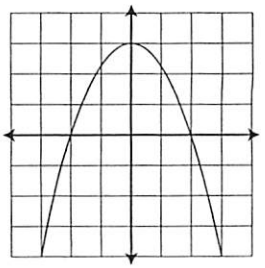


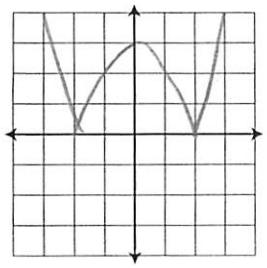
b)



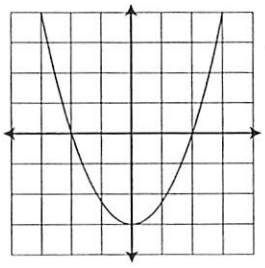


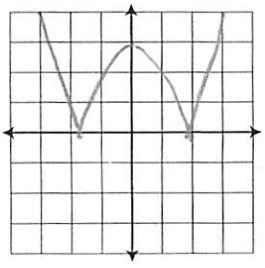
c)



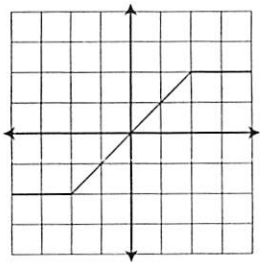


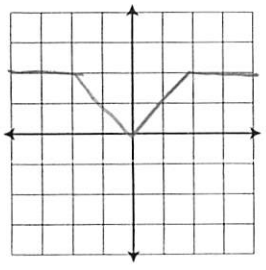
d)



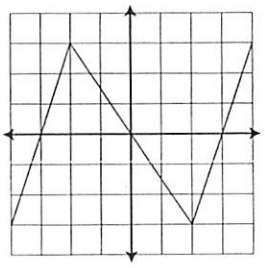


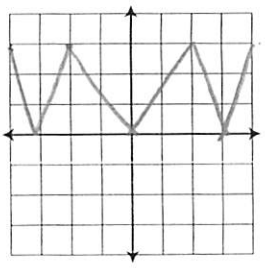
e)



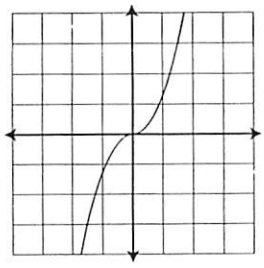


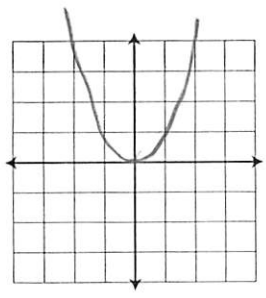
f)



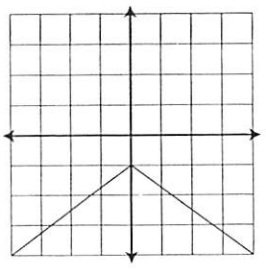


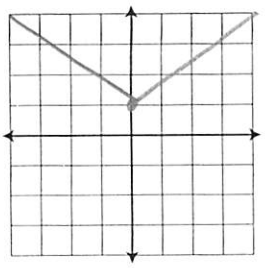
g)



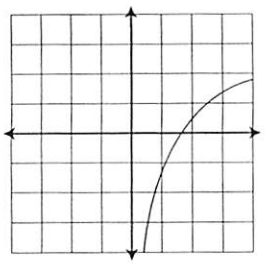


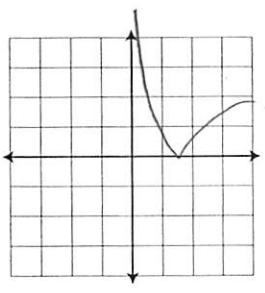
h)



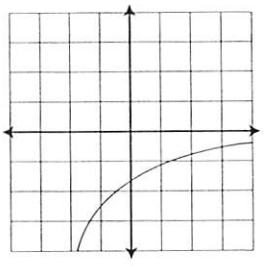


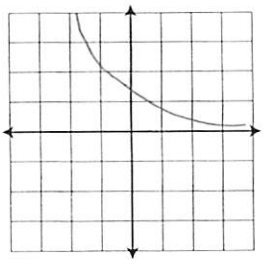
i)





j)





2. Write the absolute value function as a piecewise function.

$x+2>0 \quad x>-2$

a) $f(x)=|x+2|+1$

$x+2+1=x+3$

$-x-2+1=-x-1$

$$f(x)=\begin{cases} x+3, & x\geq -2 \\ -x-1, & x<-2 \end{cases}$$

b) $g(x)=-|x-1|+2$

c) $h(x)=\frac{1}{2}|4-x|-2$

$$h(x)=\begin{cases} -\frac{1}{2}x, & x\leq 4 \\ \frac{1}{2}x-4, & x>4 \end{cases}$$

d) $i(x)=-\frac{1}{4}|2x-4|-1$

e) $j(x)=2|x+2|-3$

$$j(x)=\begin{cases} 2x+1, & x\geq -2 \\ -2x-7, & x<-2 \end{cases}$$

f) $k(x)=-3|2-x|+4$

g) $l(x)=-\frac{3}{2}|4-6x|$

$$l(x)=\begin{cases} 9x-6, & x\leq 2/3 \\ -9x+6, & x>2/3 \end{cases}$$

h) $m(x)=-\frac{3}{2}|-6x|+4$

i) $n(x)=\frac{1}{3}|2x-1|-3$

$$n(x)=\begin{cases} \frac{2}{3}x-\frac{10}{3}, & x\geq 1/2 \\ -\frac{2}{3}x-\frac{8}{3}, & x<1/2 \end{cases}$$

j) $p(x)=-\frac{3}{4}|1-2x|+1$

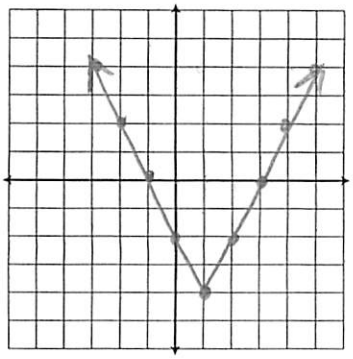
k) $q(x)=\frac{3}{4}|-2x-1|-1$

$$q(x)=\begin{cases} -\frac{3}{2}x-\frac{7}{4}, & x\leq -1/2 \\ \frac{3}{2}x-\frac{1}{4}, & x>-1/2 \end{cases}$$

l) $r(x)=|x^2-1|$

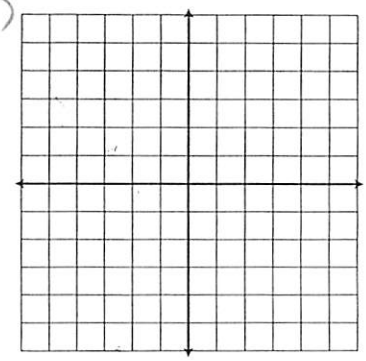
3. Graph the the absolute value function. State the intercepts, domain, and range.

a) $f(x) = 2|x-1|-4$

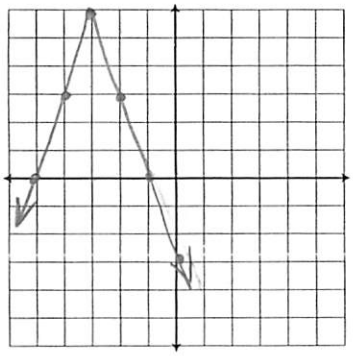


(-1,0)(3,0)
(0,-2)
D: ℝ
R: $y \geq -4$

b) $g(x) = \frac{1}{2}|x+2|+1$

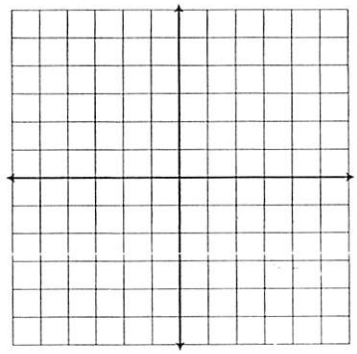


c) $h(x) = -3|x+3|+6$

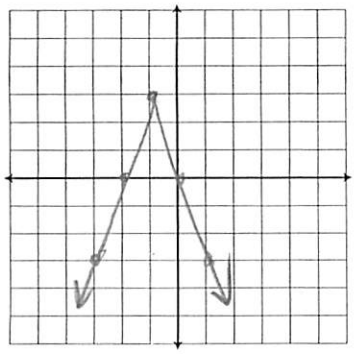


(-5,0)(-1,0)
(0,-3)
D: ℝ
R: $y \leq 6$

d) $j(x) = 2|2(1-x)|-4$

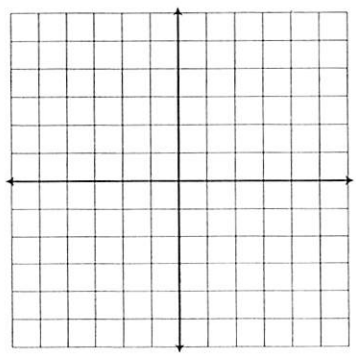


e) $j(x) = -|3(x+1)|+3$



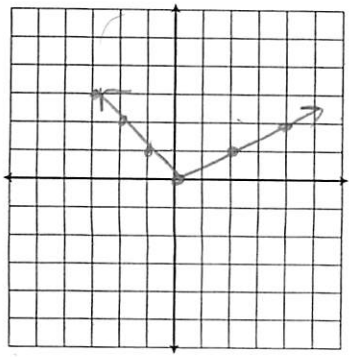
(0,0)
(-2,0)
D: ℝ
R: $y \leq 3$

f) $k(x) = -|2-x|+3$

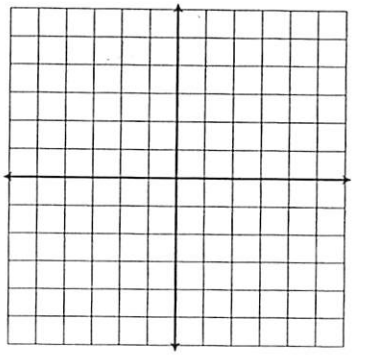


4. Graph each piecewise function.

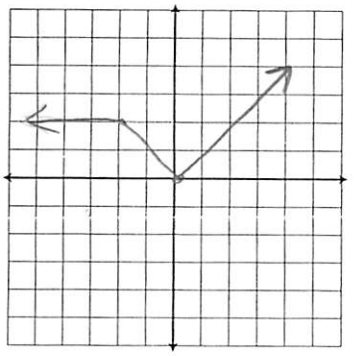
a) $f(x) = \begin{cases} \frac{1}{2}x, & x \geq 0 \\ |x|, & x < 0 \end{cases}$



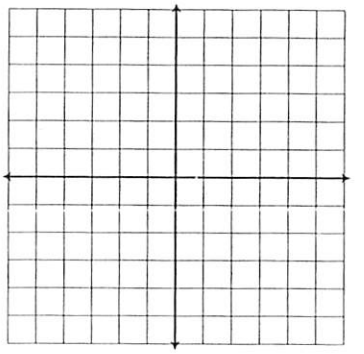
b) $g(x) = \begin{cases} |x| - 2, & x \geq -2 \\ -|x| + 2, & x < -2 \end{cases}$



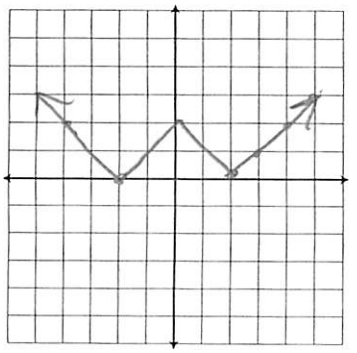
c) $h(x) = \begin{cases} 2, & x < -2 \\ |x|, & x \geq -2 \end{cases}$



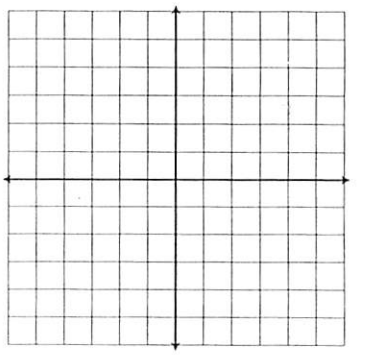
d) $i(x) = \begin{cases} |x - 4|, & x \geq 1 \\ |x + 2|, & x < 1 \end{cases}$



e) $j(x) = \begin{cases} |x - 2|, & x \geq 0 \\ |x + 2|, & x < 0 \end{cases}$

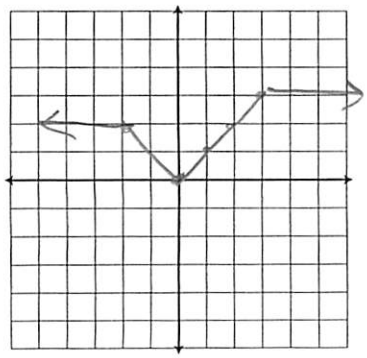


f) $k(x) = \begin{cases} -|1 - x|, & x \geq -2 \\ |x| - 5, & x < -2 \end{cases}$

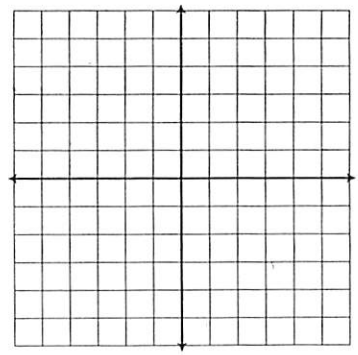


5. Graph each piecewise function.

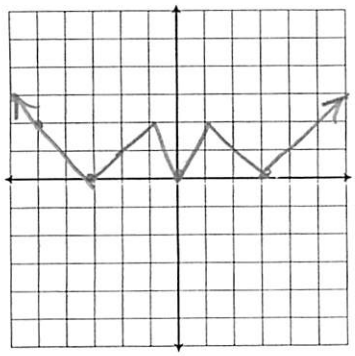
a) $f(x) = \begin{cases} 2, & x < -2 \\ |x|, & -2 \le x < 3 \\ 3, & x \ge 3 \end{cases}$



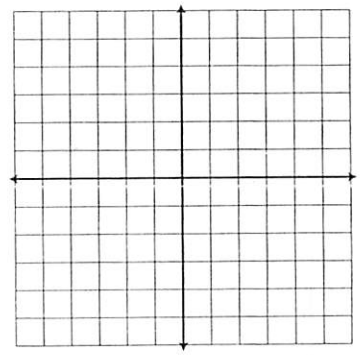
b) $g(x) = \begin{cases} x+4, & x < -2 \\ |x|, & -2 \le x < 3 \\ -x+6, & x \ge 3 \end{cases}$



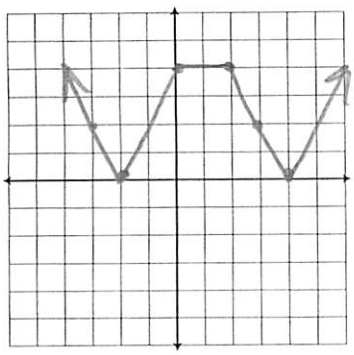
c) $h(x) = \begin{cases} |x+3|, & x < -1 \\ 2|x|, & -1 \le x < 1 \\ |x-3|, & x \ge 1 \end{cases}$



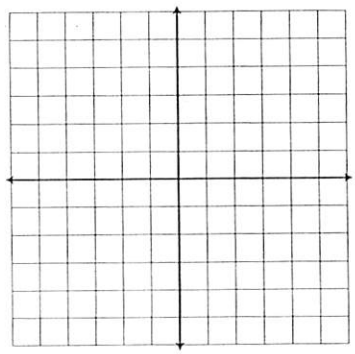
d) $i(x) = \begin{cases} -|x+4|, & x < -3 \\ -|x|+2, & -3 \le x < 3 \\ -|x-4|, & x \ge 3 \end{cases}$



e) $j(x) = \begin{cases} 2|x+2|, & x < 0 \\ 4, & 0 \le x < 2 \\ 2|x-4|, & x \ge 2 \end{cases}$



f) $k(x) = \begin{cases} |x+2|-2, & x < -1 \\ 4|x|-5, & -1 \le x \le 1 \\ |x-2|-2, & x > 1 \end{cases}$



6. If the graph of $y = f(x)$ has domain $-3 \le x \le 2$, determine the domain of $y = |f(x)|$.

$-3 \le x \le 2$

7. If the graph of $y = f(x)$ has range $-3 \le f(x) \le 2$, determine the range of $y = |f(x)|$.

8. If the point $(-2, -6)$ is on the graph of $y = f(x)$, what point is on the graph of $y = -2|f(x)| + 3$?

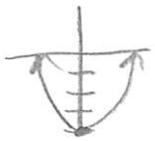
$(-2, -2|-6| + 3)$

$(-2, -2(6) + 3)$

$(-2, -9)$

9. If the point $(4, -1)$ is on the graph of $y = f(x)$, what point must be on the graph of $y = 6|f(x)|$?

10. Given $f(x) = x^2 - 4$ determine the y-intercept of $y = -2|f(x)|$.



$(0, -4)$

$(0, -2|-4|)$

$(0, -2(4))$

$(0, -8)$

11. Given $f(x) = x^2 - 9$, determine the y-intercept of $y = |f(x + 2)|$.

12. If the point $(4, -6)$ is on the graph of $y = f(x)$, what point is on the graph of $y = 12\left|\frac{1}{f(x)}\right|$?

$\left(4, \left|\frac{1}{-6}\right| \times 12\right)$

$\left(4, \left(\frac{1}{6}\right)12\right)$

$(4, 2)$

13. If the point $(-1, -2)$ is on the graph of $y = f(x)$, what point is on the graph of $y = |f(-x)|$?

4.4 Exercise Set

1. Fill in the blank.

   a) The absolute value equation $|3x - 4| = 6$ is true when $3x - 4 =$ __6__ or $3x - 4 =$ __-6__.

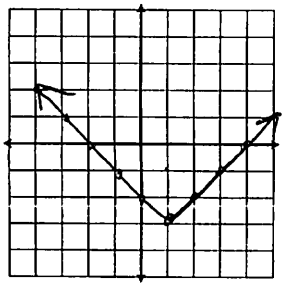
   b) The solution set $|ax + b| = c,\ c > 0$ has __2__ solutions.

   c) The solution set $|ax + b| = 0$ has __1__ solutions.

   d) The solution set $|ax + b| = c,\ c < 0$ has __0__ solutions.

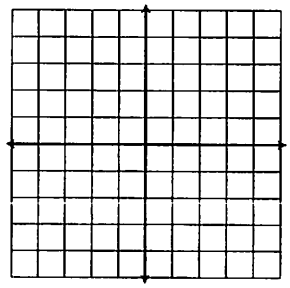
2. Graph the absolute value function and state the x-intercepts. Solve the absolute value equation.

   a) Graph: $f(x) = |x - 1| - 3$
   Solve: $|x - 1| = 3$

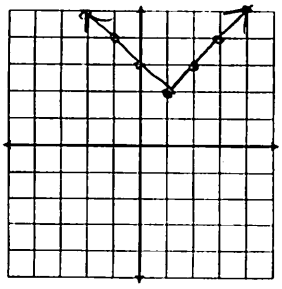


   $x = -2, 4$.

   b) Graph: $g(x) = |2x + 4|$
   Solve: $|2x + 4| = 0$

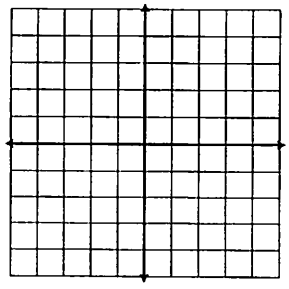


   c) Graph: $h(x) = |1 - x| + 2$
   Solve: $|1 - x| = -2$



   $\emptyset$

   d) Graph: $i(x) = -2|-x - 1| + 4$
   Solve: $-2|-x - 1| = -4$



3. Solve the absolute value equations. Check for extraneous roots.

a) $|x-4|+7=11$

$x-4=4$ $\quad$ $x-4=-4$

$x=8$ $\quad$ $x=0$

$|8-4|+7=11$ $\quad$ $|0-4|+7=11$

$4+7=11$ ✓ $\quad$ $4+7=11$ ✓

b) $|x+2|+3=10$

c) $|2y+3|+5=10$

$2y+3=5$ $\quad$ $2y+3=-5$

$2y=2$ $\quad$ $2y=-8$

$y=1$ ✓ $\quad$ $y=-4$ ✓

d) $|2y-5|-3=7$

e) $2|2z+5|+1=7$

$2(2z+5)+1=7$ $\quad$ $2(-2z-5)+1=7$

$4z+10+1=7$ $\quad$ $-4z-10+1=7$

$4z=-4$ $\quad$ $-4z=16$

$z=-1$ $\quad$ $z=-4$

f) $-3|2-z|-1=-10$

g) $-2|3-2x|+1=5$

$-2(3-2x)+1=5$

$3-2x=\frac{4}{-2}$

$3-2x=-2$

$5=2x$ $\quad$ $x=5/2$ ✗

h) $2|2-3x|+3=3$

$-2(2x-3)+1=5$

$2x-3=-2$

$2x=1$

$x=1/2$ ✗ $\quad$ ∅

i) $1.5|x-1.8|-2.4=5.1$

$|x-1.8|=5$

$x-1.8=5$ $\quad$ $x-1.8=-5$

$x=6.8$ ✓ $\quad$ $x=-3.2$ ✓

j) $\left|\frac{3}{5}y-\frac{7}{10}\right|+\frac{2}{15}=\frac{11}{15}$

k) $7=\left|\frac{3x}{5}+\frac{1}{5}\right|+2$

$5=\left|\frac{3x+1}{5}\right|$

$5=\frac{3x+1}{5}$ ✓

$25=3x+1$

$24=3x$

$x=8$ ✓

l) $5=\left|\frac{2x}{7}+\frac{4}{7}\right|-3$

$-5=\frac{3x+1}{5}$

$-25=3x+1$

$-26=3x$ $\quad$ $x=-26/3$ ✓

m) $2=-8+\left|4-\frac{1}{2}y\right|$

$10=\left|4-\frac{1}{2}y\right|$

$10=4-\frac{1}{2}y$

$6=-\frac{1}{2}y$ $\quad$ $-12=y$

$-10=4-\frac{1}{2}y$

$-14=-\frac{1}{2}y$

$28=y$

n) $1=-3+\left|2-\frac{1}{4}y\right|$

4. Solve the absolute value equations.

a) $|x+1| = 4x$

$x+1 = 4x$ $\quad$ $x+1 = -4x$

$1 = 3x$ $\quad$ $5x = -1$

$1/3 = x$ ✓ $\quad$ $x = -1/5$ ✗

b) $|3y-1| = y+5$

c) $|1-z| = 2z+3$

$1-z = 2z+3$ $\quad$ $1-z = -2z-3$

$-2 = 3z$ $\quad$ $z = -4$ ✗

$-2/3 = z$ ✓

d) $|4-3x| = 10-5x$

e) $|4x-7| = 2+x$

$4x-7 = 2+x$ $\quad$ $4x-7 = -2-x$

$3x = 9$ $\quad$ $5x = 5$

$x = 3$ ✓ $\quad$ $x = 1$ ✓

f) $|x^2-5| = 4$

++: **g)** $|3x+2| = |2x+3|$ $\quad$ +−:

$3x+2 = 2x+3$ $\quad$ $3x+2 = -2x-3$

$x = 1$ $\quad$ $5x = -5$

$x = -1$

h) $|3-x| = |x+4|$

i) $|x^2-3x-11| - 3 = 4$ ✓ 7

$x^2-3x-11 = 7$ $\quad$ $x^2-3x-11 = -7$

$x^2-3x-18 = 0$ $\quad$ $x^2-3x-4 = 0$

$(x-6)(x+3) = 0$ $\quad$ $(x-4)(x+1) = 0$

$x = 6, -3$ ✓ $\quad$ $x = 4, -1$ ✓

j) $|x^2-5x-4| = 10$

k) $|2x^2-x-12| = 9$

$2x^2-x-12 = 9$ $\quad$ $2x^2-x-12 = -9$

$2x^2-x-21 = 0$ $\quad$ $2x^2-x-3 = 0$

$(2x-7)(x+3) = 0$ $\quad$ $(2x-3)(x+1) = 0$

$x = 7/2, -3$ ✓ $\quad$ $x = 3/2, -1$ ✓

l) $|3x^2-2x-24| = 16$

5. For what value of x does each hold?

a) $|x-3|=x-3$

b) $|x+2|=x+2$

c) $|x-3|=-(x-3)$

d) $|x+2|=-(x+2)$

e) $|3x+4|=3x+4$

f) $|3x+4|=-3x-4$

g) $|x^2-9|=x^2-9$

h) $|x^2-9|=9-x^2$

i) $|x^2-x-6|=x^2-x-6$

j) $|x^2-x-6|=-(x^2-x-6)$

6. Write each of the statements as an absolute value equation.

a) x is 3 units from 4

$|x-4|=3$

b) y is 2 units from 3

c) z is 3 units from -1

$|z+1|=3$

d) a is 5 units from -3

e) $2b$ is 4 units from 3

$|2b-3|=4$

f) $3c$ is 5 units from -1

7. Joe solved the absolute value equation $|x-1|=-3$ as: $x-1=-3$ or $x-1=3$. His solution was $x=-2,4$. What was Joe's mistake?

extraneous roots

8. What is the solution to the absolute value equation $|f(x)|<0$?

9. At a high school, the highest and lowest salaries for teachers is given by the equation $|h-67\,350|=12\,150$. Find the highest and lowest salaries on the staff.

$h-67350=12150$ $h-67350=-12150$

$h=\$79,500$ $h=\$55,200$

10. For the best salmon fishing, a line must be set at d feet according to the equation $|d-65|=42$. Find the range of depth that offers the best fishing.

11. Let $N=4.7$ represent a measurement with an accuracy of 4.7 ± 0.05. Express the accuracy as an absolute value equation.

$|N-4.7|=0.05$

12. The daily production, N, of tractors is 400 ± 15. Express the highs and lows of production as an absolute value equation.

13. In a freezer, the temperature, T, must be within 3° C of −25° C. Express the highs and lows of the temperature as an absolute value equation.

$|T+25°|=3°$

14. The industrial process of a smelter has temperatures from 360° C to 420° C. Using C as the variable, write an absolute value equation that corresponds to this range.

4.5 Exercise Set

1. Which one of the following has a graph that does not have a vertical asymptote?

a) $f(x) = \frac{1}{x^2+1}$ b) $f(x) = \frac{1}{x^2-4}$ c) $f(x) = \frac{2}{x}$ d) $f(x) = \frac{3x+1}{x-4}$

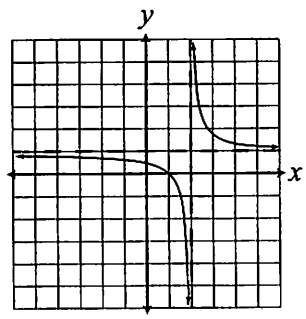
$x^2 \neq -1$

2. Which of the following has a graph that does not have a horizontal asymptote?

a) $f(x) = \frac{x+4}{2x-1}$ b) $f(x) = \frac{2x}{x^2-4}$ c) $f(x) = \frac{x^2-4}{x+2}$ d) $f(x) = \frac{x-2}{(x-1)(x+3)}$

$\frac{(x+2)(x-2)}{(x+2)}$

3. Use the graph shown to complete each statement by the directional approach method.

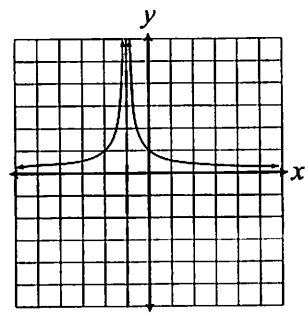


a) As $x \to -\infty$, $y \to$ 1

As $x \to \infty$, $y \to$ 1

As $x \to 2^-$, $y \to \infty$

As $x \to 2^+$, $y \to -\infty$

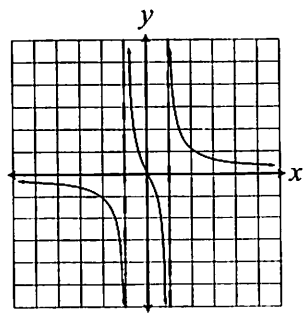


b) As $x \to -\infty$, $y \to$ 0

As $x \to \infty$, $y \to$ 0

As $x \to -1^-$, $y \to \infty$

As $x \to -1^+$, $y \to \infty$



c) As $x \to -\infty$, $y \to$ 0

As $x \to \infty$, $y \to$ 0

As $x \to -1^-$, $y \to -\infty$

As $x \to -1^+$, $y \to \infty$

As $x \to 1^-$, $y \to -\infty$

As $x \to 1^+$, $y \to \infty$

4. Find the x and y intercepts of the rational functions.

a) $y = \dfrac{x+4}{x-1}$

$0 = \dfrac{x+4}{x-1}$ $x = -4$ $(-4, 0)$

$y = \dfrac{4}{-1} = -4$ $(0, -4)$

b) $y = \dfrac{x}{x+3}$

c) $y = \dfrac{(x-1)(x+4)}{(x+2)(x-2)}$

$0 = (x-1)(x+4)$ $x = 1, -4$ $(1, 0)$ $(-4, 0)$

$y = \dfrac{(-1)(4)}{(2)(-2)} = \dfrac{-4}{-4} = 1$ $(0, 1)$

d) $y = \dfrac{3}{x+2}$

e) $y = \dfrac{1}{x^2+1}$

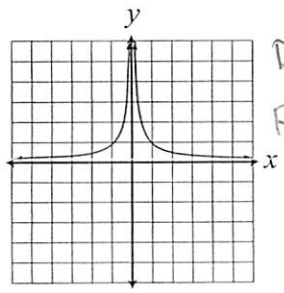
$0 = \dfrac{1}{x^2+1}$ no x-int.

$y = \dfrac{1}{1} = 1$ $(0, 1)$

f) $y = \dfrac{1}{x^2-4}$

5. Determine the domain and range of the following graphs.

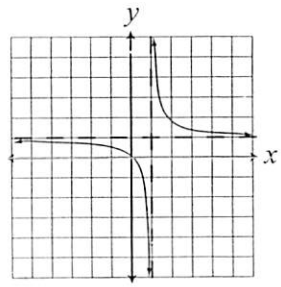
a)



D: $x \neq 0$

R: $y > 0$

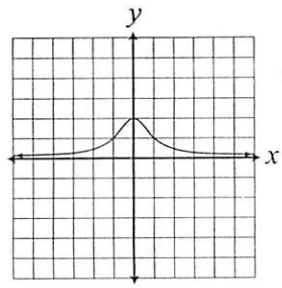
b)



D: $x \neq 1$

R: $y \neq 0$

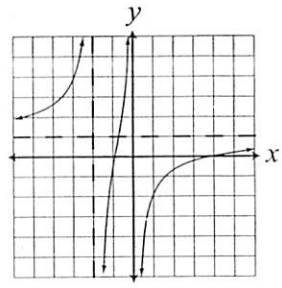
c)



D: $\mathbb{R}$

R: $y > 0$

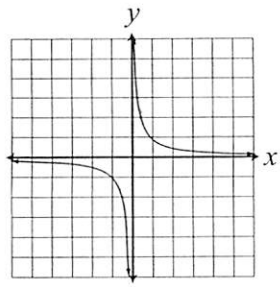
d)



D: $x \neq -2, 0$

R: $y \neq 1$

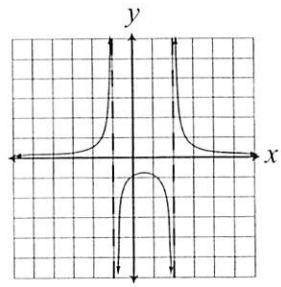
e)



D: $x \neq 0$

R: $y \neq 0$

f)



D: $x \neq -1, 2$

R: $y > 0$, $y \leq -0.8$

6. Find all asymptotes, vertical and horizontal.

a) $f(x) = \dfrac{2}{x+3}$

V: $x=-3$

H: $y=0$

b) $g(x) = \dfrac{x+4}{x+2}$

c) $h(x) = \dfrac{2}{x} - 2 = \dfrac{2-2x}{x}$

V: $x=0$

H: $y=-2$

d) $j(x) = \dfrac{1}{x+2} - 1$

$2(x+1)(x-1)$

e) $k(x) = \dfrac{2x^2-2}{x^2-4}$

$(x+2)(x-2)$

V: $x=\pm 2$.

H: $y=2$

f) $l(x) = \dfrac{3x+4}{2x-5}$

g) $m(x) = \dfrac{x-1}{x^2-1}$

$(x+1)(x-1)$

V: $x=-1$

H: $y=0$

h) $n(x) = \dfrac{-3x^2-3x+6}{x^2-9}$

$(x-4)(x+1)$

i) $p(x) = \dfrac{x^2-3x-4}{x^2+x-6}$

$(x+3)(x-2)$

V: $x=-3, 2$.

H: $y=1$

j) $q(x) = \dfrac{1}{(x-1)^2}$

k) $r(x) = \dfrac{x^2-4}{x^2+2x-3}$

$(x+3)(x-1)$

V: $x=-3, 1$

H: $y=1$

l) $s(x) = \dfrac{x^2-4x+3}{2x^2-8x}$

m) $t(x) = \dfrac{2-x}{2x^2-x-3}$

$(2x-3)(x+1)$

V: $x=3/2, -1$

H: $y=0$

n) $u(x) = \left|\dfrac{1}{x^2} - 4\right|$

o) $v(x) = \dfrac{1}{|x^2-1|}$

$(x+1)(x-1)$

V: $x=\pm 1$

H: $y=0$

p) $w(x) = \left|\dfrac{1}{x} - 2\right|$

q) $a(x) = \dfrac{1}{x^2+1}$

V: none

H: $y=0$.

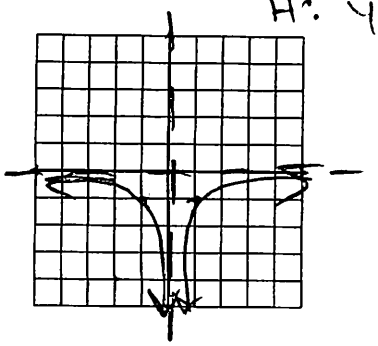
r) $b(x) = \dfrac{4-x^2}{x^2+4}$

7. Sketch the graph of the following. Label the asymptotes.

a) $f(x) = -\frac{1}{x^2}$

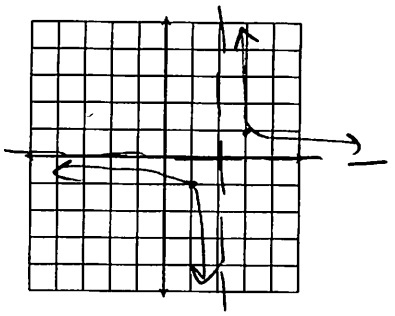
V: $x=0$

H: $y=0$



b) $g(x) = \frac{1}{x-2}$

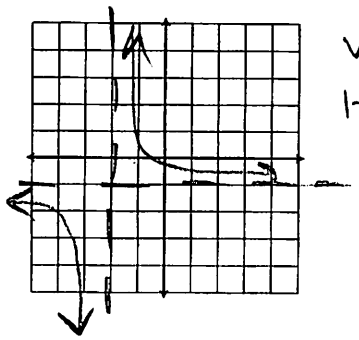
x	y
1	-1
3	1



V: $x=2$

H: $y=0$

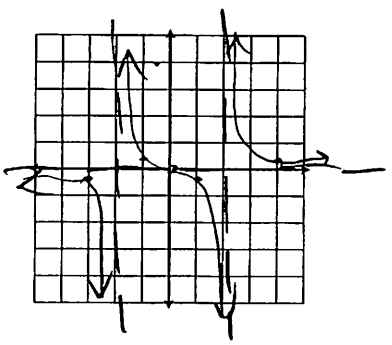
c) $h(x) = \frac{1}{x+2} - 1 = \frac{1-(x+2)}{x+2} = \frac{-x-1}{x+2}$



V: $x=-2$.

H: $y=-1$.

d) $j(x) = \frac{x}{x^2-4}$



V: $x=\pm 2$

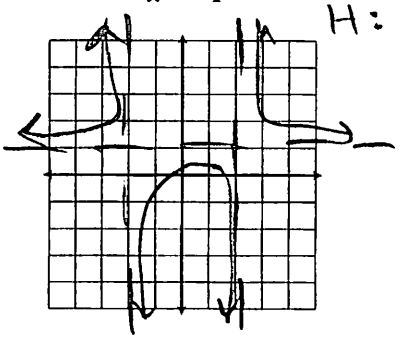
H: $y=0$.

x	y
0	0
1	$-1/3$
-1	$1/3$
4	$1/3$
-4	$-1/3$

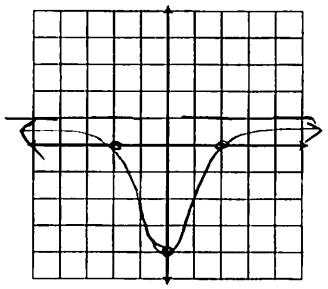
e) $k(x) = \frac{x^2-1}{x^2-4}$

V: $x=\pm 2$

H: $y=1$



f) $l(x) = \frac{x^2-4}{x^2+1}$



V: none

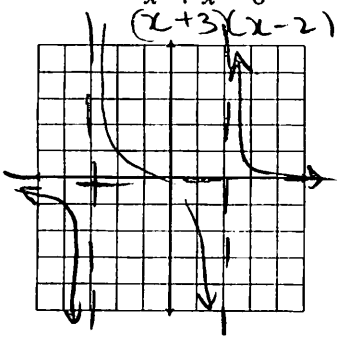
H: $y=1$.

x	y
0	-4
2	0
-2	0

g) $m(x) = \frac{x+2}{x^2+x-6}$

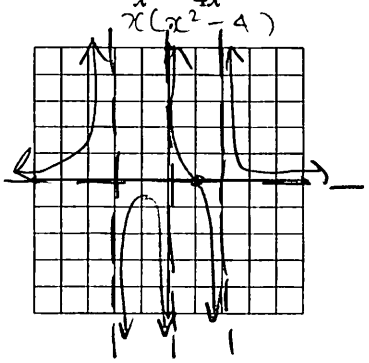
V: $x=-3, 2$

$\frac{x+2}{(x+3)(x-2)}$ H: $y=0$.



h) $n(x) = \frac{x-1}{x^3-4x}$

$x(x^2-4)$



V: $x=0, \pm 2$.

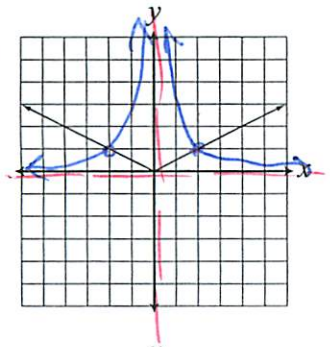
H: $y=0$

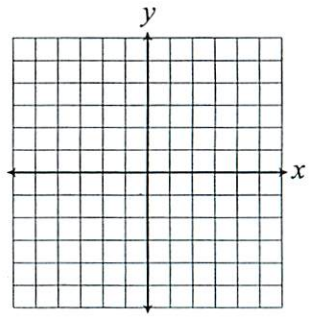
x	y
1	0

4.6 Exercise Set

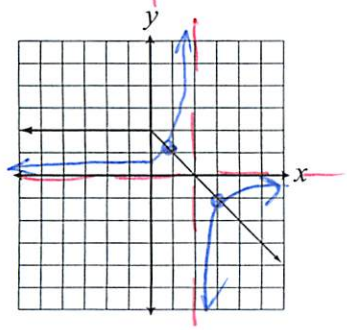
1. Given the graph of $y = f(x)$, graph $y = \dfrac{1}{f(x)}$.

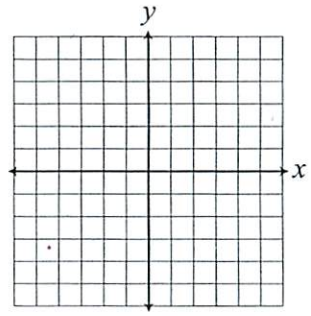
a)



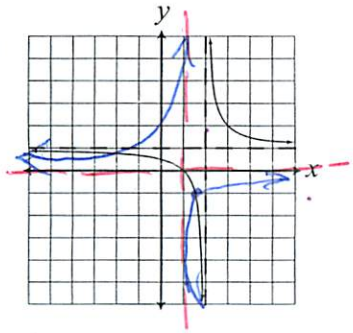


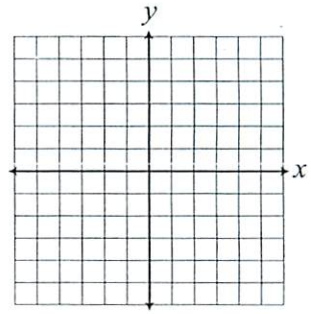
b)



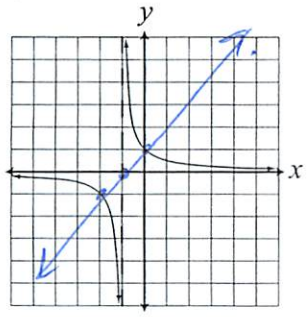


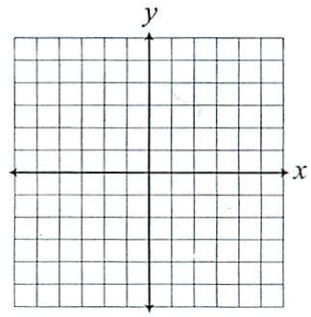
c)



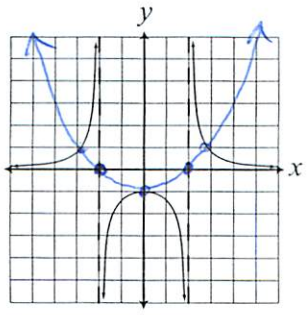


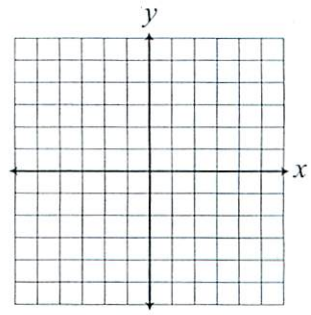
d)





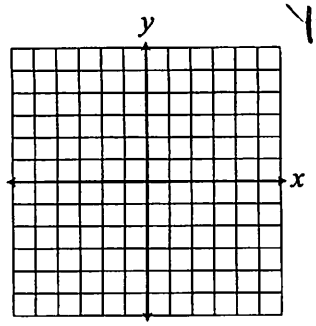
e)



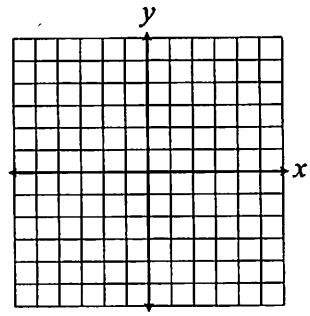


2. Find an equation of a rational function f that satisfies the given conditions.

a) Vertical asymptote $x = 1$.
Horizontal asymptote $y = -1$.
Goes through origin.



b) Vertical asymptote $x = -2$.
Horizontal asymptote $y = 1$.
x-intercept -3.



Multiple choice

3. The graph of $y = -a^x$ compared to the graph of $y = a^x$ has a:

a) reflection in the x-axis.
b) reflection in the y-axis.
c) reflection in the line $y = x$.
d) reflection in the origin.

4. If the graph of $y = f(x)$ has a zero at $x = 2$ and is undefined at $x = -4$, then $y = \frac{1}{f(x)}$ has a:

a) zero at $x = -2$ and is undefined at $x = 4$.
b) zero at $x = \frac{1}{2}$ and is undefined at $x = -\frac{1}{4}$.
c) zero at $x = 4$ and is undefined at $x = -2$.
d) zero at $x = -4$ and is undefined at $x = 2$.

5. If the point (a, b) is on the graph of $y = f(x)$, which point must be on the graph $y = \frac{1}{f(x)}$?

a) $\left(\frac{1}{a}, \frac{1}{b}\right)$
b) $\left(a, \frac{1}{b}\right)$
c) $(-a, -b)$
d) (b, a)

6. If $f(x) \geq 1$, then the reciprocal function $\frac{1}{f(x)}$ must be:

a) $\frac{1}{f(x)} \leq 1$
b) $\frac{1}{f(x)} \leq -1$
c) $0 < \frac{1}{f(x)} \leq 1$
d) $-1 \leq \frac{1}{f(x)} < 0$

7. If the graph of $y = f(x)$ has the restriction $0 < f(x) \leq 1$, then the graph of $y = \frac{1}{f(x)}$ will have the restriction:

a) $\frac{1}{f(x)} \geq 1$

b) $\frac{1}{f(x)} \leq -1$

c) $0 < \frac{1}{f(x)} \leq 1$

d) $-1 \leq \frac{1}{f(x)} < 0$

8. If the point $(6, -3)$ is on the graph of $y = f(x)$, what point is on the graph $y = 3\left|\frac{1}{f(x)}\right|$?

a) $(2, -1)$

b) $(2, 1)$

c) $(6, 1)$

d) $(18, 1)$

9. If the graph of $y = f(x)$ has the restriction $-2 \leq f(x) \leq 2$, then the reciprocal graph $y = g(x)$ will have which one of the following restrictions?

a) $g(x) \leq -\frac{1}{2},\ g(x) \geq \frac{1}{2}$

b) $g(x) \geq -\frac{1}{2},\ g(x) \leq \frac{1}{2}$

c) $g(x) \leq -\frac{1}{2},\ g(x) \leq \frac{1}{2}$

d) $g(x) \leq -2,\ g(x) \geq 2$

10. Given the function $f(x) = 3x + 1$, which of the following will have the same y-intercept as $f(x)$?

a) $y = -\frac{1}{f(x)}$

b) $y = \frac{1}{f(x)}$

c) $x = f(y)$

d) $x = \frac{1}{f(y)}$

11. If the point (a, b) is on the graph of $y = f(x)$, which point is on the graph $y = -\frac{1}{f(x+1)}$, $a \neq 0, b \neq 0$?

a) $(a + 1, -b)$

b) $(a - 1, -b)$

c) $\left(a + 1, -\frac{1}{b}\right)$

d) $\left(a - 1, -\frac{1}{b}\right)$

Solve using a graphing calculator.

12. The average cost per unit of producing widgets is $C(x) = \frac{1}{5}x + 2 + \frac{2000}{x}$, where x is the number of widgets produced. At what production level will the average cost be minimal?

13. The average cost per year of a computer is given by $C(x) = \frac{2500}{x} + 175 + 25x$, where x is the age of the computer in years. When is the average cost per year a minimum?

14. A sleeping pill is taken at 10:00 pm. The strength of the pill is given by $S(t) = 8t/(t^2 + 1)$, where t is the number of hours since taking the pills. At what time is the greatest effect of the sleeping pill felt?

15. A new product is put on the market. Sales are modelled by $S(t) = 150t/(t^2 + 100)$, where $S(t)$ is the daily sales in thousands t days after the product is released. After how many days did sales reach a maximum, and what was the maximum sales?

16. What is the smallest possible value for the sum of a positive number and its reciprocal?

17. The product of two positive numbers is 10. For what values of the numbers will the sum be the smallest?

4.7 Chapter Review

Section 4.1

1. Factor completely.

a) $x^2 - 10x + 24$

$(x-6)(x-4)$

b) $x^2 - 4x - 45$

c) $x^4 - 16$

$(x^2+4)(x^2-4)$

$(x^2+4)(x+2)(x-2)$

d) $x^2 - xy - 12y^2$

e) $50 - 5y - y^2$

$-(y^2+5y-50)$

$-(y+10)(y-5)$

f) $3x^2 - 12$

g) $-x^2 - y^2$

$-(x^2+y^2)$

h) $4x^3 - 12x^2y + 8xy^2$

i) $3x^2 - 6x$

$3x(x-2)$

j) $x^2y^2 + 7xy + 12$

k) $-2x^6 + 8x^5 - 8x^4$

$-2x^4(x^2-4x+4)$

$-2x^4(x-2)^2$

l) $x^4y - 81y^5$

2. Find all integer values of k such that:

a) $x^2 + kx + 10$ factors.

$1 \cdot 10 \quad \pm 11$

$2 \cdot 5 \quad \pm 7$

b) $x^2 + kx - 10$ factors.

c) $x^2 + 3x + k$ factors. $|k| < 20$

d) $x^2 - 2x + k$ factors. $|k| < 20$

Section 4.2

3. Factor completely.

a) $2x^2 + 11x + 12$

$(2x+3)(x+4)$

b) $6x^2 - 17x - 3$

c) $8x^2 + 18x - 5$

$(4x-1)(2x+5)$

d) $9x^3 + 12x^2 - 45x$

e) $x^3 + 3x^2 - 9x - 27$

$x^2(x+3) - 9(x+3)$

$(x^2-9)(x+3)$

$(x+3)(x-3)(x+3)$.

f) $-12 - x^2y^2 - 8xy$

g) $27x^2 - 144xy + 192y^2$

h) $6(x-1)^2 + 7y(x-1) - 3y^2$

i) $18x^3y + 3x^2y^2 - 6xy^3$

j) $2x(x-3) + (x-1)(x+2)$

4. Find all integer values of k such that:

a) $2x^2 + kx - 3$ factors.

b) $2x^2 - 5x + k$ factors. $|k| < 20$

Section 4.3

5. Write the absolute value function as a piecewise function.

a) $f(x) = -2|1 - 2x| - 3$

$y = -2(1-2x) - 3$

$y = -2 + 4x - 3$

$y = 4x - 5. \quad x \leq 1/2$

$y = -2(2x-1) - 3$

$y = -4x - 1 \quad x > 1/2.$

b) $g(x) = \frac{1}{2}|4x - 3| + 1$

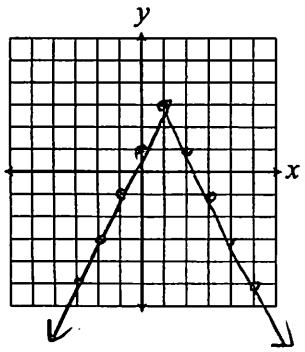
6. Graph the absolute value function.

$-2+2x+3$ $y=2x+1$ $x\leq 1$

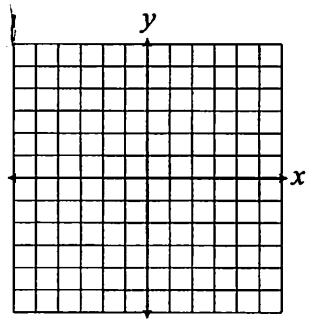
a) $h(x)=-2|1-x|+3$

$-2x+2+3$

$y=-2x+5$ $x>1$



b) $i(x)=\begin{cases}|x-1|, & x\geq -1\\ 3|x+2|-1, & x<-1\end{cases}$



Section 4.4

7. Solve.

a) $|4x+1|=3$

$4x+1=3$

$4x=2$

$x=1/2$

$4x+1=-3$

$4x=-4$

$x=-1$

b) $|2x-1|=3-x$

c) $|x+1|=-2x$ ← can't be negative!

$x+1=-2x$

$\varnothing$.

d) $|2x^2+x-12|=9$

8. For what value of x is each statement true?

a) $|2x-1|=1-2x$

$|2x-1|=1-2x$

$2x-1=1-2x$

$4x=2$

$x=1/2$

$-2x+1=1-2x$

$x\leq 1/2$

b) $|-3x-1|=3x+1$

$x\geq -1/3$

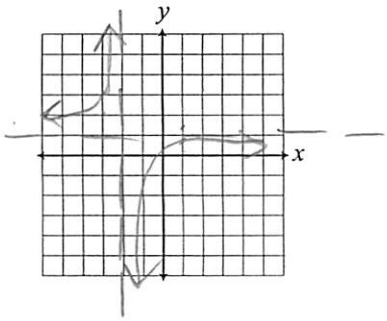
Section 4.5

9. $f(x)=\dfrac{(x+2)(x-1)}{x-1}$ is a linear equation with the point $x=1$, $y=3$ missing.

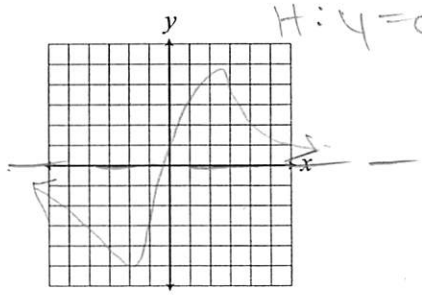
$(1,3)$

10. Sketch the graph of the following. Label the asymptotes.

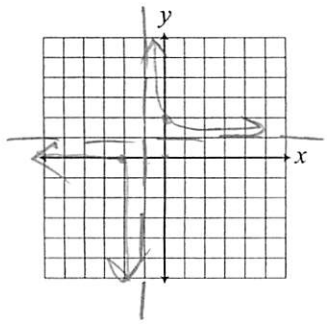
a) $f(x) = \dfrac{x}{x+2}$ H: $y=1$. V: $x=-2$



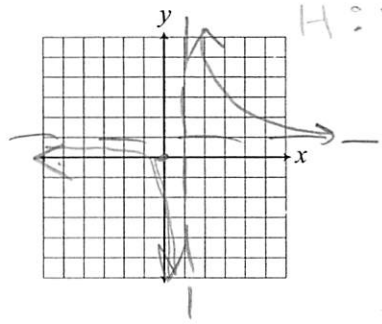
b) $g(x) = \dfrac{8x}{x^2+1}$ no V. H: $y=0$



c) $h(x) = \dfrac{x^2-4}{x^2-x-2}$ $\dfrac{(x+2)(x-2)}{(x-2)(x+1)}$ V: $x=-1$ hole when $x=2$. $(2, 4/3)$ H: $y=1$



d) $i(x) = \dfrac{x^2}{(x-1)^2}$ V: $x=1$. H: $y=1$.



Section 4.6

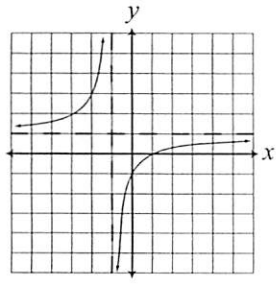
11. When $f(x) \le -1$, then $\dfrac{1}{f(x)}$ is $-1 \le \dfrac{1}{f(x)} < 0$.

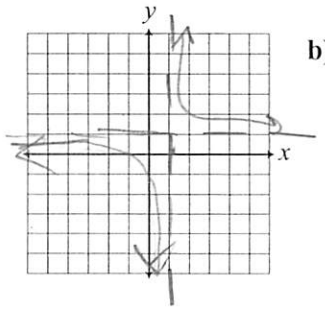
12. When $f(x)$ is undefined, $\dfrac{1}{f(x)}$ is 0.

13. When $0 < f(x) \le 1$, $\dfrac{1}{f(x)}$ is ≥ 1.

14. Given the graph $y = f(x)$, graph $y = \dfrac{1}{f(x)}$.

a)





b)